Abstract Algebra, Number Theory, Statistics

A Review of Research Activity[1]

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Introduction

Current: Teaching Fellow in Mathematics and Statistics

- **Lead/Design** modules, and supervise projects, for Data Analytics MSc and Mathematics BSc.
- **Research** in Algebra, Number Theory, and Statistics.

Past: Academia and Industry

- Pricing Analyst/R Programmer, **ERV Travel Insurance**.
- Postdoctoral Research Fellow, **University of Edinburgh**.
- PhD, Algebra and Number Theory, **University of Exeter**.
- Research Assistant, Statistical Modelling, **University of Exeter and Plymouth Marine Lab**.
- MMath, First Class, Pure Mathematics and Statistics, **University of Exeter**.

Research Interests: Pure Mathematics and Statistics

Algebra and Number Theory

- **Abstarct algebra** with applications in **number theory** and **mathematical physics**.
- **Classification** of two kinds of algebraic objects: **skew braces** and **Hopf-Galois structures**.
- During PhD **classified**, for the first time, all skew braces and Hopf-Galois Structures of degree p^3 for a prime p in [NZ18].
- PhD examined and **passed** with **no correction**.
- Two (out of six) chapters were enhanced and published **in Journal of Algebra** [NZ19], a Q1 Journal among journals for algebra and number theory.
- Currently working on **further two publications**.

Yang-Baxter Equation

Skew braces provide solutions to a fundamental equation in mathematical physics, the Yang-Baxter equation.

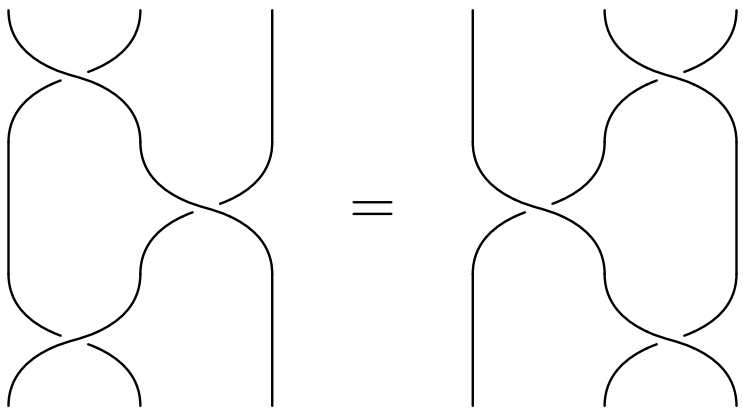
For a vector space $\mathcal{V}$ an element

$$R \in \mathrm{GL}(\mathcal{V} \otimes \mathcal{V})$$

is said to satisfy the **Yang-Baxter equation (YBE)** if

$$(R \otimes I)(I \otimes R)(R \otimes I) = (I \otimes R)(R \otimes I)(I \otimes R)$$

holds. This equation can be "*depicted*" by



The equation was first introduced in **statistical mechanics** during 1970s and has since appeared in many other areas: **knot theory**, **tensor categories**, ...

Hopf-Galois Theory

Hopf-Galois structures encode information relating to the structure of the **rings of integers** of **extensions** of the rational numbers $\mathbb{Q}$.

- For L/K a **Galois extension** of fields with **Galois group** G.
- **Normal basis theorem:** action of Hopf algebra $K[G]$ on L turns L into a free $K[G]$-module of rank one.
- Hopf-Galois structures are similar to $K[G]$ they are K-**Hopf algebras** together with an **action** on L.
- **Question:** How can we find all Hopf-Galois structures for L/K when $[L : K] = n$?
- **Answer: difficult** for general n. For $n = pq, p, p^2$ and a few other cases the problems is solved.
- Finding Hopf-Galois structures **help** us understand the **structure** of $\mathcal{O}_L$, the **ring of integers** of L, as modules.

Theorem (A Summary of Results)

The number of Hopf-Galois structures on L/K with Galois group G of size p^3 is

G	$e(G)$
C_{p^3}	p^2
$C_{p^2} \times C_p$	$(2p-1)p^3$
C_p^3	$(p^4+2p^3-p-1)p^3$
$C_p^2 \rtimes C_p$	$(2p^2+p-2)p^3$
$C_{p^2} \rtimes C_p$	$(2p-1)p^3$

where $p > 3$ is a prime number.

Proof.

Consists of 150 pages of intricate group theoretic calculations in **"On Hopf-Galois Structures and Skew Braces of Order p^3"** [cf. NZ18]. □

Remarks

Hopf-Galois structures are **parametrised** by skew braces, and so we find **all solutions** of the **Yang-Baxter equation** with **dimension** of $\mathcal{V}$ equal to p^3.

Calculation for 5^{th} row, together with **automorphism groups of skew braces**, were published in the Journal of Algebra [cf. NZ19].

A **generalisation** of the 6^{th} row, and number theoretic applications, is **work in progress**....

Interest in the results are growing...
Invited to speaker at the **University of Nebraska, Omaha, U.S.** and **Keele University, UK** this summer.

Research Interest II

Statistics

- **Statistical modelling** and data analysis.
- **Collaborative** work with **industry and other academic** institutions.
- Joint with scientists at Plymouth Marine Lab **analysed a large data set** containing NASA's satellite estimations of ocean colour.
- Matched estimation with in situ data and designed a statistical model to **understand the uncertainty**.
- Results, published in **Journal of Remote Sensing**, demonstrate a model that **explains** 67% of the squared error as a potentially correctable bias [cf. NZELCB$^+$18].

Thank You for Your Attention!

Selected Publications:

[NZ18] Kayvan Nejabati Zenouz. On Hopf-Galois Structures and Skew Braces of Order p^3. *The University of Exeter, PhD Thesis, Funded by EPSRC DTG*, January 2018. https://ore.exeter.ac.uk/repository/handle/10871/32248.

[NZ19] Kayvan Nejabati Zenouz. Skew Braces and Hopf-Galois Structures of Heisenberg Type. *Journal of Algebra*, 524:187–225, April 2019. https://doi.org/10.1016/j.jalgebra.2019.01.012.

[NZELCB+18] Kayvan Nejabati Zenouz, Peter E. Land, Trevor C. Bailey, Malcolm Taberner, Silvia Pardo, Shubha Sathyendranath, Vicki Brammall, Jamie D. Shutler, and Graham D. Quartly. A Statistical Modeling Framework for Characterising Uncertainty in Large Datasets: Application to Ocean Colour. *Remote Sensing*, 10, May 2018. https://doi.org/10.3390/rs10050695.

Research Statement:

www.nejabatiz.com/Files/Research.pdf